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Equivalence between dual-phase-lagging and two-phase-system heat conduction processes

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Abstract

We show the equivalence between the dual-phase-lagging heat conduction and the Fourier heat conduction in two-phase systems subject to lack of local thermal equilibrium. This provides an additional tool for studying the two heat-conduction processes and shows the possibility of thermal oscillation and resonance in two-phase-system heat conduction, a phenomenon observed experimentally. Such thermal waves and possibly resonance come from the macroscale coupled conductive terms.

1. Introduction

By lumping microstructural effects into delayed temporal responses in the macroscopic formulation, Tzou [1] proposed a dual-phase-lagging constitutive equation for heat conduction that relates the temperature gradient ∇T at a material point $\vec{x}$ and time $t+\tau_T$ to the heat flux density vector $\mathbf{q}$ at $\vec{x}$ and time $t+\tau_q$ through material thermal conductivity k,

$$\mathbf{q}(\vec{x}, t+\tau_q) = -k\nabla T(\vec{x}, t+\tau_T). \tag{1}$$

Two delay times τ_T and τ_q are regarded as intrinsic thermal or structural properties of the material. The former is due to the microstructural interactions such as phonon–electron interaction or phonon scattering, and is termed as the phase-lag of the temperature gradient. The latter is, on the other hand, interpreted as the relaxation time accounting for the fast-transient effects of thermal inertia, and is named as the phase-lag of the heat flux.

Xu and Wang [2] built the relationship between the constitutive model (1) and the Boltzmann transport equation. With the constitutive model (1), the first law of thermodynamics leads to delay/advanced dual-phase-lagging heat-conduction equations [2]. Xu and Wang [2] discussed proper initial and boundary conditions and developed analytical solutions of such equations. Also obtained in [2] were the conditions under which thermal oscillations occur.

Expanding ∇T and $\mathbf{q}$ with respect to time t by Taylor series and retaining only the first-order terms in τ_T and τ_q, we obtain a linear version of (1) at point $\vec{x}$ and time t [1,3]

$$\mathbf{q} + \tau_q \frac{\partial \mathbf{q}}{\partial t} = -k\left[\nabla T + \tau_T \frac{\partial}{\partial t}(\nabla T)\right], \tag{2}$$

which is known as the Jeffreys-type constitutive equation of heat flux [4]. Eliminating $\mathbf{q}$ from Eq. (2) and the classical energy equation leads to the dual-phase-lagging heat-conduction equation that reads, if all thermophysical material properties are assumed to be constant,

$$\frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2} = \alpha \Delta T + \alpha \tau_T \frac{\partial}{\partial t}(\Delta T) + \frac{\alpha}{k}\left[S(\vec{x}, t) + \tau_q \frac{\partial S(\vec{x}, t)}{\partial t}\right], \tag{3}$$

where α is the thermal diffusivity of the material, Δ is the Laplacian, and S stands for the volumetric internal heat sources.

The dual-phase-lagging heat-conduction equation forms a generalized, unified equation with the classical parabolic

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heat-conduction equation, the hyperbolic heat-conduction equation, the energy equation in the phonon scattering model [4,5], and the energy equation in the phonon–electron interaction model [6–8] as its special cases [3,9,10]. This, with its success in describing and predicting phenomena such as ultrafast pulse-laser heating, propagating of temperature pulses in superfluid liquid helium, nonhomogeneous lagging response in porous media, thermal lagging in amorphous materials, and effects of material defects and thermomechanical coupling [3,11], has given rise to the research effort on various aspects of dual-phase-lagging heat conduction.

The dual-phase-lagging heat conduction was shown to be admissible by the Boltzmann transport equation [2] and by the second law of the extended irreversible thermodynamics [3]. It was also proven to be well-posed in a finite region of n-dimensions ($n \geqslant 1$) under any linear boundary conditions including Dirichlet, Neumann and Robin types [12,13]. Solutions of one-dimensional (1D) heat conduction were obtained for some specific initial and boundary conditions in [1,11,14–18]. Wang and Zhou [9,19] and Wang et al. [10] developed methods of measuring the phase-lags of the heat flux and the temperature gradient and obtained analytical solutions for regular 1D, 2D and 3D heat-conduction domains under essentially arbitrary initial and boundary conditions. The solution structure theorems were also developed for both mixed and Cauchy problems of dual-phase-lagging heat-conduction equations [9,10,12] by extending those for the hyperbolic heat conduction [20]. These theorems build relationships among the contributions (to the temperature field) by the initial temperature distribution, the source term and the initial time-rate change of the temperature, uncover the structure of temperature field and considerably simplify the development of solutions. Xu and Wang [21] addressed thermal features of the dual-phase-lagging heat-conduction, conditions and features of thermal oscillation and resonance and their difference with those in the classical and hyperbolic heat conductions in particular. They show that both the underdamped oscillation and the critically-damped oscillation cannot appear if the phase lag of the temperature gradient τ_T is larger than that of the heat flux τ_q. The modes of underdamped thermal oscillation are limited to a region fixed by two relaxation distances defined by $\sqrt{\alpha\tau_T}\left(\sqrt{\frac{\tau_q}{\tau_T}}+\sqrt{\frac{\tau_q}{\tau_T}-1}\right)$ and $\sqrt{\alpha\tau_T}\left(\sqrt{\frac{\tau_q}{\tau_T}}-\sqrt{\frac{\tau_q}{\tau_T}-1}\right)$ for the case of $\tau_T>0$, and by one relaxation distance $2\sqrt{\alpha\tau_q}$ for the case of $\tau_T=0$. Here α is the thermal diffusivity of the medium. Tzou [3] and Vadasz [22–25] developed an *approximate* equivalence between the heat conduction in porous media and the dual-phase-lagging heat conduction, and applied the latter to examine features of the former.

Based on the equivalence, Vadasz [22–25] shows that τ_T is always larger than τ_q in porous-medium heat conduction so that thermal waves cannot occur according to the necessary condition for thermal waves in dual-phase-lagging heat conduction [21]. However, such waves are observed in casting sand experiments by two independent groups [3]. In an attempt to resolve this difference and to build the intrinsic relationship between the two heat-conduction processes, we present an *exact* equivalence between the dual-phase-lagging heat conduction and the Fourier heat conduction in two-phase systems (including porous media) subject to lack of local thermal equilibrium. Based on this new equivalence, we also show the possibility of and uncover the mechanism responsible for the thermal oscillation in two-phase-system heat conduction.

2. Heat conduction in two-phase systems

The microscale model for heat conduction in two-phase systems is well-known. It consists of field equation and constitutive equation. The field equation comes from the conservation of energy (the first law of thermodynamics). The commonly-used constitutive equation is the Fourier law of heat conduction for the relation between the temperature gradient ∇T and the heat flux density vector $\mathbf{q}$ [26].

For transport in two-phase systems, the macroscale (so-called Darcy scale in the literature) is a phenomenological scale that is much larger than the microscale of pores and grains and much smaller than the system length scale. The interest in the macroscale rather than the microscale comes from the fact that a prediction at the microscale is complicated because of complex microscale geometry of two-phase systems such as porous media, and that we are usually more interested in large scales of transport for practical applications. Existence of such a macroscale description equivalent to the microscale behavior requires a good separation of length scale and has been well discussed in [27].

To develop a macroscale model of transport in two-phase systems, the method of volume averaging starts with a microscale description [28,29]. Both conservation and constitutive equations are introduced at the microscale. Resulting microscale field equations are then averaged over a representative elementary volume (REV), the smallest differential volume resulting in statistically meaningful local average properties, to obtain macroscale field equations. In the process of averaging, *multiscale theorems* are used to convert integrals of gradient, divergence, curl, and partial time derivatives of a function to some combination of gradient, divergence, curl, and partial time derivatives of integrals of the function and integrals over the boundary of the REV [28,29]. The readers are referred to [28,29] for the details of the method of volume averaging and to [28] for a summary of the other methods of obtaining macroscale models.

Quintard and Whitaker [30] use the method of volume averaging to develop one- and two-equation macroscale models for heat conduction in two-phase systems. First, they define the microscale problem by the first law of thermodynamics and the Fourier law of heat conduction (Fig. 1)

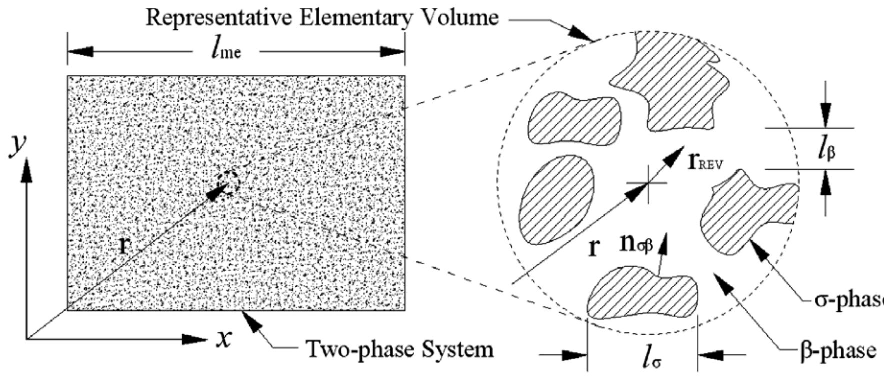

Fig. 1. Rigid two-phase system.

$$(\rho c)_\beta \frac{\partial T_\beta}{\partial t} = \nabla \cdot (k_\beta \nabla T_\beta), \quad \text{in the } \beta\text{-phase} \tag{4}$$

$$(\rho c)_\sigma \frac{\partial T_\sigma}{\partial t} = \nabla \cdot (k_\sigma \nabla T_\sigma), \quad \text{in the } \sigma\text{-phase} \tag{5}$$

$$T_\beta = T_\sigma, \quad \text{at the } \beta - \sigma \text{ interface } A_{\beta\sigma} \tag{6}$$

$$\mathbf{n}_{\beta\sigma} \cdot k_\beta \nabla T_\beta = \mathbf{n}_{\beta\sigma} \cdot k_\sigma \nabla T_\sigma, \quad \text{at the } \beta - \sigma \text{ interface } A_{\beta\sigma} \tag{7}$$

Here ρ, c and k are the density, specific heat and thermal conductivity, respectively, subscripts β and σ refer to the β- and σ-phases, respectively, $A_{\beta\sigma}$ represents the area of the β–σ interface contained in the REV; $\mathbf{n}_{\beta\sigma}$ is the outward-directed surface normal from the β-phase toward the σ-phase, and $\mathbf{n}_{\beta\sigma} = -\mathbf{n}_{\sigma\beta}$ (Fig. 1). To be complete, Quintard and Whitaker [30] have also specified the initial conditions and the boundary conditions at the entrances and exits of the REV; however, we need not do so for our discussion.

Next Quintard and Whitaker [30] apply an averaging process with multiscale theorems [31] to Eqs. (4) and (5) to obtain a two-equation model

$$\epsilon_\beta (\rho c)_\beta \frac{\partial \langle T_\beta \rangle^\beta}{\partial t} = \nabla \cdot \{\mathbf{K}_{\beta\beta} \cdot \nabla \langle T_\beta \rangle^\beta + \mathbf{K}_{\beta\sigma} \cdot \nabla \langle T_\sigma \rangle^\sigma\} + h a_v (\langle T_\sigma \rangle^\sigma - \langle T_\beta \rangle^\beta), \tag{8}$$

and

$$\epsilon_\sigma (\rho c)_\sigma \frac{\partial \langle T_\sigma \rangle^\sigma}{\partial t} = \nabla \cdot \{\mathbf{K}_{\sigma\sigma} \cdot \nabla \langle T_\sigma \rangle^\sigma + \mathbf{K}_{\sigma\beta} \cdot \nabla \langle T_\beta \rangle^\beta\} - h a_v (\langle T_\sigma \rangle^\sigma - \langle T_\beta \rangle^\beta), \tag{9}$$

where ϵ_β and ϵ_σ are the volume fractions of the β- and σ-phases with $\epsilon_\beta = \varphi$, $\epsilon_\sigma = 1 - \varphi$ and a constant porosity φ for a rigid two-phase system; h and a_v come from modeling of the interfacial flux and are the film heat transfer coefficient and the interfacial area per unit volume, respectively; $\mathbf{K}_{\beta\beta}$, $\mathbf{K}_{\sigma\sigma}$, $\mathbf{K}_{\beta\sigma}$ and $\mathbf{K}_{\sigma\beta}$ are the effective thermal conductivity tensors, and the coupled thermal conductivity tensors are equal

$\mathbf{K}_{\beta\sigma} = \mathbf{K}_{\sigma\beta}$.

$$\langle T_\beta \rangle^\beta = \frac{1}{V_\beta} \int_{V_\beta} T_\beta \mathrm{d}V, \tag{10}$$

and

$$\langle T_\sigma \rangle^\sigma = \frac{1}{V_\sigma} \int_{V_\sigma} T_\sigma \mathrm{d}V, \tag{11}$$

where V_β and V_σ are the volumes of β-phase and σ-phase in REV, respectively.

When the system is isotropic and physical properties of the two phases are constant, Eqs. (8) and (9) reduce to

$$\gamma_\beta \frac{\partial \langle T_\beta \rangle^\beta}{\partial t} = k_\beta \Delta \langle T_\beta \rangle^\beta + k_{\beta\sigma} \Delta \langle T_\sigma \rangle^\sigma + h a_v (\langle T_\sigma \rangle^\sigma - \langle T_\beta \rangle^\beta), \tag{12}$$

and

$$\gamma_\sigma \frac{\partial \langle T_\sigma \rangle^\sigma}{\partial t} = k_\sigma \Delta \langle T_\sigma \rangle^\sigma + k_{\sigma\beta} \Delta \langle T_\beta \rangle^\beta - h a_v (\langle T_\sigma \rangle^\sigma - \langle T_\beta \rangle^\beta), \tag{13}$$

where $\gamma_\beta = \varphi(\rho c)_\beta$ and $\gamma_\sigma = (1-\varphi)(\rho c)_\sigma$ are the β-phase and σ-phase effective thermal capacities, respectively, φ is the porosity, k_β and k_σ are the effective thermal conductivities of the β- and σ-phases, respectively, and $k_{\beta\sigma} = k_{\sigma\beta}$ is the cross effective thermal conductivity of the two phases.

The one-equation model is valid whenever the two temperatures $\langle T_\beta \rangle^\beta$ and $\langle T_\sigma \rangle^\sigma$ are sufficiently close to each other so that

$$\langle T_\beta \rangle^\beta = \langle T_\sigma \rangle^\sigma = \langle T \rangle. \tag{14}$$

This *local thermal equilibrium* is valid when any one of the following three conditions occurs [29,30]: (1) either ϵ_β or ϵ_σ tends to zero, (2) the difference in the β-phase and σ-phase physical properties tends to zero, (3) the square of the ratio of length scales $(l_{\beta\sigma}/L)^2$ tends to zero (e.g. steady, one-dimensional heat conduction). Here $l_{\beta\sigma}^2 = [\epsilon_\beta \epsilon_\sigma (\epsilon_\beta k_\sigma + \epsilon_\sigma k_\beta)]/(h a_v)$, and $L = L_T L_{T1}$ with L_T and L_{T1} as the characteristic lengths of $\nabla \langle T \rangle$ and $\nabla\nabla \langle T \rangle$, respectively, such that $\nabla \langle T \rangle = O(\Delta \langle T \rangle / L_T)$ and $\nabla\nabla \langle T \rangle = O(\Delta \langle T \rangle / L_{T1} L_T)$.

When the local thermal equilibrium is valid, Quintard and Whitaker [30] add Eqs. (8) and (9) to obtain a one-equation model

$$\langle\rho\rangle C\frac{\partial\langle T\rangle}{\partial t}=\nabla\cdot[\mathbf{K}_{\text{eff}}\cdot\nabla\langle T\rangle]. \tag{15}$$

Here $\langle\rho\rangle$ is the spatial average density defined by

$$\langle\rho\rangle=\epsilon_\beta\rho_\beta+\epsilon_\sigma\rho_\sigma, \tag{16}$$

and C is the mass-fraction-weighted thermal capacity given by

$$C=\frac{\epsilon_\beta(\rho c)_\beta+\epsilon_\sigma(\rho c)_\sigma}{\epsilon_\beta\rho_\beta+\epsilon_\sigma\rho_\sigma}. \tag{17}$$

The effective thermal conductivity tenor is

$$\mathbf{K}_{\text{eff}}=\mathbf{K}_{\beta\beta}+2\mathbf{K}_{\beta\sigma}+\mathbf{K}_{\sigma\sigma}. \tag{18}$$

The choice between the one-equation model and the two-equation model has been well discussed in [29,30]. They have also developed methods of determining the effective thermal conductivity tensor $\mathbf{K}_{\text{eff}}$ in the one-equation model and the four coefficients $\mathbf{K}_{\beta\beta}$, $\mathbf{K}_{\beta\sigma}=\mathbf{K}_{\sigma\beta}$, $\mathbf{K}_{\sigma\sigma}$, and ha_{v} in the two-equation model. Their studies suggest that the coupling coefficients are on the order of the smaller of $\mathbf{K}_{\beta\beta}$ and $\mathbf{K}_{\sigma\sigma}$. Therefore, the coupled conductive terms should not be omitted in any detailed two-equation model of heat conduction processes. When the principle of local thermal equilibrium is not valid, the commonly-used two-equation model in the literature is the one without the coupled conductive terms [32]

$$\epsilon_\beta(\rho c)_\beta\frac{\partial\langle T_\beta\rangle^\beta}{\partial t}=\nabla\cdot(\mathbf{K}_{\beta\beta}\cdot\nabla\langle T_\beta\rangle^\beta)+ha_{\text{v}}(\langle T_\sigma\rangle^\sigma-\langle T_\beta\rangle^\beta), \tag{19}$$

and

$$\epsilon_\sigma(\rho c)_\sigma\frac{\partial\langle T_\sigma\rangle^\sigma}{\partial t}=\nabla\cdot(\mathbf{K}_{\sigma\sigma}\cdot\nabla\langle T_\sigma\rangle^\sigma)-ha_{\text{v}}(\langle T_\sigma\rangle^\sigma-\langle T_\beta\rangle^\beta). \tag{20}$$

On the basis of the above analysis, we now know that the coupled conductive terms $\mathbf{K}_{\beta\sigma}\cdot\nabla\langle T_\sigma\rangle^\sigma$ and $\mathbf{K}_{\sigma\beta}\cdot\nabla\langle T_\beta\rangle^\beta$ cannot be discarded in the exact representation of the two-equation model. However, we could argue that Eqs. (19) and (20) represent a reasonable approximation of Eqs. (8) and (9) for a heat conduction process in which $\nabla\langle T_\beta\rangle^\beta$ and $\nabla\langle T_\sigma\rangle^\sigma$ are *sufficiently close* to each other. Under these circumstances $\mathbf{K}_{\beta\beta}$ in Eq. (19) would be given by $\mathbf{K}_{\beta\beta}+\mathbf{K}_{\beta\sigma}$ while $\mathbf{K}_{\sigma\sigma}$ in Eq. (20) should be interpreted as $\mathbf{K}_{\sigma\beta}+\mathbf{K}_{\sigma\sigma}$. This limitation of Eqs. (19) and (20) is believed to be the reason behind the paradox of heat conduction in porous media subject to lack of local thermal equilibrium well-analyzed in [33]. For an isotropic system with constant physical properties of the two phases, Eqs. (19) and (20) reduce to the traditional formulation of heat conduction in two-phase systems [33–35]

$$\gamma_\beta\frac{\partial\langle T_\beta\rangle^\beta}{\partial t}=k_{e\beta}\Delta\langle T_\beta\rangle^\beta+ha_{\text{v}}(\langle T_\sigma\rangle^\sigma-\langle T_\beta\rangle^\beta), \tag{21}$$

and

$$\gamma_\sigma\frac{\partial\langle T_\sigma\rangle^\sigma}{\partial t}=k_{e\sigma}\Delta\langle T_\sigma\rangle^\beta-ha_{\text{v}}(\langle T_\sigma\rangle^\sigma-\langle T_\beta\rangle^\beta), \tag{22}$$

where we introduce the *equivalent* effective thermal conductivities $k_{e\beta}=k_\beta+k_{\beta\sigma}$ and $k_{e\sigma}=k_\sigma+k_{\sigma\beta}$ for the β- and σ-phases, respectively, to take the above note into account. To describe the thermal energy exchange between solid and gas phases in casting sand, Tzou [3] has also directly postulated Eqs. (21) and (22) (using k_β and k_σ rather than $k_{e\beta}$ and $k_{e\sigma}$) as a two-step model, parallel to the two-step equations in the microscopic phonon–electron interaction model [6–8].

3. Equivalence between two heat-conduction processes

We develop an equivalence between the dual-phase-lagging and two-phase-system heat conductions based on Eqs. (12) and (13). We first rewrite Eqs. (12) and (13) in their operator form

$$\begin{bmatrix}\gamma_\beta\frac{\partial}{\partial t}-k_\beta\Delta+ha_{\text{v}} & -k_{\beta\sigma}\Delta-ha_{\text{v}}\\ -k_{\beta\sigma}\Delta-ha_{\text{v}} & \gamma_\sigma\frac{\partial}{\partial t}-k_\sigma\Delta+ha_{\text{v}}\end{bmatrix}\begin{bmatrix}\langle T_\beta\rangle^\beta\\ \langle T_\sigma\rangle^\sigma\end{bmatrix}=0. \tag{23}$$

We then obtain a uncoupled form by evaluating the operator determinant such that

$$\left[\left(\gamma_\beta\frac{\partial}{\partial t}-k_\beta\Delta+ha_{\text{v}}\right)\left(\gamma_\sigma\frac{\partial}{\partial t}-k_\sigma\Delta+ha_{\text{v}}\right)-\left(k_{\beta\sigma}\Delta+ha_{\text{v}}\right)^2\right]\langle T_i\rangle^i=0, \tag{24}$$

where the index i can take β or σ. Its explicit form reads, after dividing by $ha_{\text{v}}(\gamma_\beta+\gamma_\sigma)$

$$\frac{\partial\langle T_i\rangle^i}{\partial t}+\tau_q\frac{\partial^2\langle T_i\rangle^i}{\partial t^2}=\alpha\Delta\langle T_i\rangle^i+\alpha\tau_T\frac{\partial}{\partial t}(\Delta\langle T_i\rangle^i)+\frac{\alpha}{k}\left[S(\vec{x},t)+\tau_q\frac{\partial S(\vec{x},t)}{\partial t}\right], \tag{25}$$

where

$$\tau_q=\frac{\gamma_\beta\gamma_\sigma}{ha_{\text{v}}(\gamma_\beta+\gamma_\sigma)},\quad \tau_T=\frac{\gamma_\beta k_\sigma+\gamma_\sigma k_\beta}{ha_{\text{v}}(k_\beta+k_\sigma+2k_{\beta\sigma})},$$
$$k=k_\beta+k_\sigma+2k_{\beta\sigma},\quad \alpha=\frac{k}{\rho c}=\frac{k_\beta+k_\sigma+2k_{\beta\sigma}}{\gamma_\beta+\gamma_\sigma}, \tag{26}$$
$$S(\vec{x},t)+\tau_q\frac{\partial S(\vec{x},t)}{\partial t}=\frac{k_{\beta\sigma}^2-k_\beta k_\sigma}{ha_{\text{v}}}\Delta^2\langle T_i\rangle^i.$$

Therefore, $\langle T_\beta\rangle^\beta$ and $\langle T_\sigma\rangle^\sigma$ satisfy *exactly* the same dual-phase-lagging heat-conduction equation (Eqs. (3) and (25)). The reported conductivity and diffusivity data of two-phase systems (nanofluids, bi-composite media, porous media etc.) in the literature were based on the Fourier heat conduction and should be reexamined.

Note that Eqs. (12) and (13) are the mathematical representation of the first law of thermodynamics and the Fourier law of heat conduction for heat conduction processes in two-phase systems at the macroscale. Therefore, we have an *exact* equivalence between dual-phase-lagging heat conduction and Fourier heat conduction in two-phase systems. This is significant because all results in these two fields become mutually applicable. In particular, all analytical methods and results (such as the solution structure theorems) in [9,10,19] can be applied to study heat conduction in two-phase systems such as nanofluids, bi-composite media and porous media.

By Eq. (26), we can readily obtain that, in the two-phase-system heat conduction

$$\frac{\tau_T}{\tau_q} = 1 + \frac{\gamma_\beta^2 k_\sigma + \gamma_\sigma^2 k_\beta - 2\gamma_\beta\gamma_\sigma k_{\beta\sigma}}{\gamma_\beta\gamma_\sigma(k_\beta + k_\sigma + 2k_{\beta\sigma})}. \quad (27)$$

It can be larger, equal or smaller than 1 depending on the sign of $\gamma_\beta^2 k_\sigma + \gamma_\sigma^2 k_\beta - 2\gamma_\beta\gamma_\sigma k_{\beta\sigma}$. Therefore, by the condition for the existence of thermal waves in [21], we may have thermal waves in two-phase-system heat conduction when $\gamma_\beta^2 k_\sigma + \gamma_\sigma^2 k_\beta - 2\gamma_\beta\gamma_\sigma k_{\beta\sigma} < 0$. Note also that for heat conduction in two-phase systems, there is a time-dependent source term S in the dual-phase-lagging heat conduction (Eqs. (25) and (26)). Therefore, the resonance can also occur in two-phase systems. This agrees with the experimental data of casting sand tests in [3]. Discarding the coupled conductive terms in Eqs. (12) and (13) assumes $k_{\beta\sigma} = 0$ so that τ_T/τ_q is always larger than 1, which leads to the exclusion of thermal oscillation and resonance [22–25] and generates an inconsistency between theoretical and experimental results in the literature regarding the possibility of thermal waves and resonance in two-phase-system heat conduction [3,22–25]. The coupled conductive terms in Eqs. (12) and (13) are thus responsible for the thermal waves and resonance in two-phase-system heat conduction subject to lack of local thermal equilibrium. These thermal waves and possibly resonance are believed to be the driving force for the extraordinary conductivity enhancement reported in nanofluids and porous media [36–38].

Although each of τ_T and τ_q is ha_v-dependent, their ratio τ_T/τ_q is not; this makes its evaluation much simpler as detailed in [22].

4. Concluding remarks

An exact equivalence exists between dual-phase-lagging heat conduction and Fourier heat conduction in two-phase systems subject to lack of local thermal equilibrium. This builds the intrinsic relationship between the two processes, reconciles the inconsistency between theoretical and experimental results in the literature, and enables us to apply the methods and results in one field to another. Due to the coupled conduction of the two phases, thermal waves and possibly resonance may appear in two-phase-system heat conduction subject to lack of local thermal equilibrium, a phenomenon observed experimentally.

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